Question 1 (7 marks)

From a random sample of people, it was found that 54 of them subscribe to a streaming music service. A symmetric confidence interval for the true population proportion who subscribe is .

(a) Determine the value of , by first finding the mid-point of the interval. (3 marks)

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| **Solution** |
|  |
| **Specific behaviours** |
| ✓ calculates mid-point  ✓ writes equation using mid-point for  ✓ determines |

(b) Determine the confidence level of the interval. (4 marks)

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| **Solution** |
| Standard error:      Hence a 95% confidence interval |
| **Specific behaviours** |
| ✓ calculates standard error  ✓ uses interval formula  ✓ determines z-score  ✓ states confidence level |

Question 2 (9 marks)

The management at a conference centre was concerned about the quality of the free pens that it provided in its meeting rooms. A staff member tested a random sample of 150 pens and found that 18 of them fail to write.

(a) If is the true proportion of pens that fail to write and is the corresponding sample proportion, use the above sample to determine

(i) . (1 mark)

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| **Solution** |
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| **Specific behaviours** |
| ✓ calculates |

(ii) the approximate margin of error for a 98% confidence interval for . (3 marks)

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| **Solution** |
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| **Specific behaviours** |
| ✓ calculates z-score  ✓ calculates standard error  ✓ calculates margin of error |

(iii) an approximate 98% confidence interval for . (1 mark)

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| **Solution** |
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| **Specific behaviours** |
| ✓ evaluates interval |

(b) The stationery company that supplies pens to the conference centre claim that no more than 3 in 50 pens fail to write. Use your previous working to comment on the validity of this claim. (2 marks)

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| **Solution** |
| .  The interval calculated in (a) contains 0.06 and so the claim is valid. |
| **Specific behaviours** |
| ✓ compares proportion to confidence interval.  ✓ states claim is valid |

(c) Comment on how the margin of error would change in (a) (ii) if

(i) the quality of the pens had been better. (1 mark)

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| **Solution** |
| Decrease, as is further from 0.5. |
| **Specific behaviours** |
| ✓ states change |

(ii) the required level of confidence decreased. (1 mark)

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| **Solution** |
| Decrease, as z-score lower. |
| **Specific behaviours** |
| ✓ states change |

Question 3 (8 marks)

A student planned to investigate what proportion of the 1260 students at their school had access to more than one computer at home.

(a) The student thought of the following three ways to select a sample from the population. Briefly discuss the main source of bias in each method.

(i) Wait at the bus-bay after school and ask the first 50 students who show up.

(1 mark)

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| **Solution** |
| Biased towards students who catch bus. |
| **Specific behaviours** |
| ✓ identifies group bias |

(ii) Advertise the survey in a whole school assembly and ask the first 50 students who volunteer to stay behind. (1 mark)

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| **Solution** |
| Self-selected samples are likely to suffer from non-response bias. |
| **Specific behaviours** |
| ✓ identifies self-selection bias |

(iii) Select and ask every 100th student from the school roll. (1 mark)

|  |
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| **Solution** |
| Small samples likely to be biased - in this case sample of only 13. |
| **Specific behaviours** |
| ✓ identifies small sample bias |

(b) Assuming that 80% of students had access to more than one computer at home, the student carried out 100 simulations in which a sample proportion was calculated from a random sample of 64 students.

(i) Explain why it is reasonable to expect that the distribution of the sample proportions would approximate normality. (2 marks)

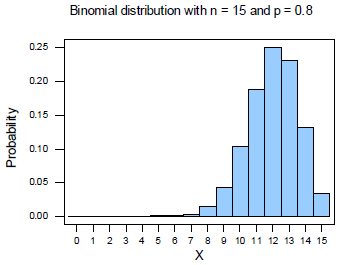
|  |
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| **Solution** |
| The sample size of 64 is reasonably large ().  Also, both and exceed the rule-of thumb minimum of 10. |
| **Specific behaviours** |
| ✓ states large sample size  ✓ indicates dependence on both and |

(ii) Determine the mean and standard deviation of the normal distribution that the sample proportions would approximate. (2 marks)

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| **Solution** |
| Mean of  Standard deviation of |
| **Specific behaviours** |
| ✓ states mean  ✓ states standard deviation |

4. (7 marks)

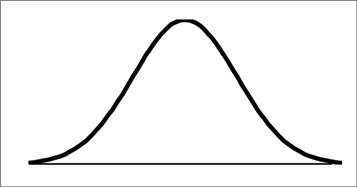
(a) (i) Sketch a histogram of a binomial distribution with . (1)



Accept any histogram skewed as above. ✓ (1)

(ii) Sketch a graph of the distribution of the sample means when 20 samples

of size 30 are drawn from a population that has a mean of 10. (2)



10

Accept any tightly clustered normal distribution about 10. ✓ ✓ (2)

(b) Two students took random samples from the Year 11 girls at the local

high school and noted the number of girls that had blonde hair.

Student 1 sampled 10 students and found 2 blondes.

Student 2 sampled 100 students and found 23 blondes.

(i) Which student would be expected to have the best indication of the

proportion of blondes in Year 11? Explain. (2)

Student 2 as his sample is much larger. ✓ ✓

A third student, Student 3, took 10 different samples of 10 students and

counted the number of blondes in each sample. He then averaged the

averages of his samples and got 2.56.

(ii) Which of the three students would have had the best prediction result?

Explain. (2)

Student 3 would have had the best prediction result as the mean of the

sampling distribution is very close to the mean. ✓ ✓

Question 5 (8 marks)

A student repeatedly took random samples of size from a large population in which it was known that of people were left-handed. For each sample, the proportion of left-handed people was calculated and recorded as the sample proportion.

(a) Use an appropriate binomial distribution to determine the probability that the sample proportion is no more than in a randomly chosen sample. (3 marks)

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| **Solution** |
|  |
| **Specific behaviours** |
| ✓ states parameters   indicates least number of successes   correct probability |

(b) After recording a large number of sample proportions, the student used them to create a histogram from which the approximate normality of their distribution was evident.

(i) Determine the expected mean and standard deviation of the observed normal distribution. (2 marks)

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| **Solution** |
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| **Specific behaviours** |
| ✓ correct mean   correct sd |

(ii) Use this normal distribution to determine the probability that the sample proportion is no more than in a randomly chosen sample. (1 mark)

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| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct probability |

(iii) Describe how the parameters calculated in (i) would change if the student took larger random samples. (2 marks)

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| **Solution** |
| Mean would stay the same.  SD would decrease. |
| **Specific behaviours** |
| ✓ states no change in mean   states decrease in sd |